Numerical Treatment of Singular BVPs in ODEs

Ewa B. Weinmüller

We consider boundary value problems for systems in ODEs which exhibit singular points in the interval of integration. These problems typically have the form

$$\begin{aligned} &z'(t) = 1/t^{\alpha} f(t, z(t)), \quad t \in (0, 1], \\ &g(z(0), z(1)) = 0, \end{aligned}$$

where α is positive. Depending on the value of α one distinguishes between week singularity, $\alpha < 1$, singularity of the first kind, $\alpha = 1$, and essential singularity, $\alpha > 1$. Last year, we discussed the analytical properties of such problems, especially the existence and uniqueness of bounded solutions, an important prerequisite for the well-posedness of the system. We also pointed out typical difficulties arising in convergence theory for standard discretization methods.

In the first talk, we shortly recall the basic analytical concepts, stability of the involved nonlinear operator and the notion of isolated solution. Moreover, we show how they relate to each other. We will also develop the basic concepts for the discretization methods in context of regular problems. Sufficient conditions for the convergence of the discretization and of the Newton iteration will be specified.

In the two following talks, we will investigate the convergence of the forward Euler scheme and polynomial collocation applied to a certain subclass of the above BVP problems with a singularity of the first kind.