Analysis of Two Viscoplastic Frictionless Contact Problems

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Abstract

We consider two quasistatic problems which describe the contact between a viscoplastic body and an obstacle, the so-called foundation. The contact is frictionless and is modelled with normal compliance of such a type that the penetration is not restricted in the first problem, but is restricted with unilateral constraint, in the second one. For each problem we derive a variational formulation, then we prove its unique solvability. The proofs are based on arguments of history-dependent quasivariational inequalities. Next, we prove the convergence of the weak solution of the first problem to the weak solution of the second problem, as the stiffness coefficient of the foundation converges to infinity. Finally, we provide a numerical validation of this convergence result. To this end we introduce fully discrete schemes for the numerical approximation of the contact problems, implement them on a computer code and present numerical simulation results in the study of a two-dimensional example.

Histrory-dependent Variational Inequalities

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Abstract

We consider a class of quasivariational inequalities which involve a special type of operators, the so-called *history-dependent operators.* We prove an existence and uniqueness result of the solution, then we complete it with a regularity result. The proofs are based on arguments of monotonicity, convexity, and fixed point. Next, we present a penalization method in the study of such inequalities. To this end, we start by introducing the penalized problem, then we prove its unique solvability as well as the convergence of its solution to the solution of the original problem, as the penalization parameter converges to zero. Finally, we present a concrete example of variational inequality for which our results work. It concerns a mathematical model which describes the quasistatic contact between a viscoelastic body and a foundation; the material's behavior is modelled with a constitutive law with long memory; the contact is frictionless and is modelled with a multivalued normal compliance condition and unilateral constraint.

Primal and Dual Variational Inequalities in Contact Mechanics

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Abstract

A large number of mathematical models in Contact Mechanics lead to variational inequalities. For part of these inequalities the unknown is either the displacement or the velocity field. These inequalities are referred in the literature as *primal variational formulations* and are used extensively in order to provide the analysis and the numerical approximation of the corresponding contact problems. Nevertheless, for a large number of contact problems it is possible to derive variational formulations expressed in term of the stress, the so-called *dual variational formulations*. In this lecture we consider a general class of history-dependent variational inequalities which represent the primal variational formulation associated to a generic frictional contact model. We associate at each inequality of this class a new history-dependent variational inequality which represents the dual variational formulation for the corresponding contact model. Then we provide the variational analysis of these inequalities including existence, uniqueness and equivalence result. The proofs are based on arguments of monotonicity, convexity, and fixed point. Finally, we show how these abstract results can be used in the study of representative mathematical models of contact with elastic and viscoelastic materials.