SHAPE OPTIMIZATION AND FREE BOUNDARY PROBLEMS

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ABSTRACT. Free boundary problems are challenging from a theoretical as well as numerical point of view. Efficient numerical solution strategies can be built on an equivalent formulation as a shape optimization problem. We briefly recall some aspects of shape optimization and discuss the shape gradient for a shape optimization problem related to the Bernoulli free boundary problem. We present a technique which is based on the material derivative of the state. For the derivation of material derivative we consider an approach based on the implicit function theorem as well as a technique using the weak formulation of the state equation.

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SHAPE DERIVATIVES WITHOUT TAKING THE SHAPE DERIVATIVE

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ABSTRACT. In the first lecture we carried out a sensitivity analysis for a shape optimization problem related to the Bernoulli free boundary problem (BFBP). A fundamental ingredient in this analysis was the material derivative of the states which however had to be eliminated in order to render the shape gradient of the cost in the form predicted by the Delfour-Hadamard-Zolesio structure theorem. In this lecture, therefore, we reconsider the same shape optimization problem and demonstrate that by an appropriate reformulation of the cost functional the use of the material derivative of the states can be replaced by their mere Hölder continuous dependence on the deformation parameter. Due to the special structure of the cost functional the shape gradient for the Dirichlet-Neumannn formulation of the BFBP does not involve an adjoint state. We therefore apply the same technique to another formulation of the BFBP tracking the Dirichlet condition on the free boundary. Finally we extract from these examples an abstract framework which allows to compute the shape gradient for a wide class of shape optimization problems subject to a PDE constraint bypassing the use of the material / shape derivatives of the states.

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