MECHANICS OF RANDOM MEDIA I

Martin Ostoja-Starzewski
Department of Mechanical Science and Engineering,
Institute for Condensed Matter Theory, and Beckman Institute
University of Illinois at Urbana-Champaign, USA

Microstructural randomness is present in most solid materials, both natural and man-made. When the separation of scales does not hold, various concepts of deterministic continuum solid mechanics need to be re-examined and new methods developed. In this talk we first review basic concepts of classifying and methods of simulation of random microstructures. Then, we turn to the scaling from a Statistical Volume Element (SVE) to a Representative Volume Element (RVE). The key role is played here by the Hill-Mandel condition (1963) which has to be chosen according to a type of material (elastic versus inelastic…) and type of loading (static, incremental, dynamic…). The RVE is set up in terms of two hierarchies of bounds stemming, respectively, from Dirichlet and Neumann boundary value problems defined for the SVE. Within this framework, we introduce the concept of a *scaling function* that describes "finite-size scaling" of thermally conducting or elastic crystalline aggregates. While the finite size is represented by the *mesoscale*, the scaling function depends on an appropriate measure quantifying the single-crystal anisotropy. We demonstrate these concepts with the scaling of the fourth-rank elasticity and the second-rank thermal conductivity tensors. We also discuss the trends in approaching the RVE for planar conductivity, linear/non-linear (thermo)elasticity, elasto-plasticity, and Darcy permeability of random media.

MECHANICS OF RANDOM MEDIA II

Martin Ostoja-Starzewski
Department of Mechanical Science and Engineering,
Institute for Condensed Matter Theory, and Beckman Institute
University of Illinois at Urbana-Champaign, USA

The methodology of Part I forms the basis for a wide range of topics in stochastic mechanics: construction of classical and non-classical continuum models of random media, optimal structures in the presence of randomness, geodesic shear bands in plastic flow, and tensor random fields (TRFs). While the mathematics of scalar RFs is well developed, the tensor case poses many challenges. We first discuss the basic properties of TRFs of a wide-sense stationary (WWS) kind, with particular focus on isotropic correlation functions with generally anisotropic realizations. The key role is played here by the theory of invariants and the group theory. Next, we examine the consequences for WSS fields of stress, strain, rotation, curvature-torsion, and couple-stress. As the second application of TRFs, we study what models may arise if they are to represent constitutive responses, leading us to stochastic finite elements (SFE) – as opposed to deterministic finite elements – where an element is smaller than the RVE. A paradigm for use of SVE arises in the wavefront propagation in random media, where, in general, a wavefront's thickness is smaller than the size of RVE of deterministic continuum mechanics.

FRACTALS IN MECHANICS OF MATERIALS

Martin Ostoja-Starzewski
Department of Mechanical Science and Engineering,
Institute for Condensed Matter Theory, and Beckman Institute
University of Illinois at Urbana-Champaign, USA

Over the years, various mechanisms of morphogenesis of fractals in diverse physical systems have been identified: liquid streams meandering over landscapes, diffusion-limited aggregation, percolation of microcracks in elastic-brittle solids, formation of shear-bands in rocks, or formation of plastic ridges in ice fields. Here we discuss the morphogenesis of fractals in the context of elastic-inelastic transitions in solids and soils. Suppose we monotonically load a randomly inhomogeneous, non-fractal elastic-plastic material from zero stress level beyond the plastic limit, then fractal patterns of plasticized grains are found to gradually form in the material domain as the plasticity fills the entire domain, and the sharp kink in the stress-strain curve is replaced by a smooth change. By analogy to the scaling analysis of phase transitions in condensed matter physics, we recognize the fully plastic state as a critical point where the critical exponents are universal regardless of the randomness in various constitutive properties and their random noise levels.

The second topic concerns the modeling of fractal porous media by continuum mechanics using the method of dimensional regularization. The basis of this method is to express the balance laws for fractal media in terms of fractional integrals and, then, to convert them to integer-order integrals in conventional (Euclidean) space. Following an account of this method, we develop balance laws of fractal media (continuity, linear and angular momenta, energy, and second law) and discuss wave equations in several settings (1d and 3d wave motions, fractal Timoshenko beam, and elastodynamics under finite strains). In all the cases, the derived equations for fractal media depend explicitly on fractal dimensions and reduce to conventional forms for continuous media with Euclidean geometries upon setting the dimensions to integers.

THERMOELASTICITY WITH FINITE WAVE SPEEDS

Martin Ostoja-Starzewski
Department of Mechanical Science and Engineering,
Institute for Condensed Matter Theory, and Beckman Institute
University of Illinois at Urbana-Champaign, USA

Hyperbolic thermoelasticity is a vital area of research in continuum thermomechanics, free of the classical paradox of infinite propagation speeds of thermal signals in Fourier-type heat conduction. Several models were developed over the past four decades, the two leading theories involving one or two relaxation times. [The theory with one relaxation time has its roots in the Maxwell-Cattaneo model of heat conduction in a rigid conductor.] While the resulting field equations are linear partial differential ones, the complexity of both theories is due to the coupling of mechanical with thermal fields. We review some mathematical aspects of the theories and solution methods for initial/boundary value problems. We also discuss some physical aspects of hyperbolic thermoelasticity such as: the consequence of Galilean invariance; various continuum thermodynamics models as starting points for the derivation of constitutive laws; helices and chiral media in homogeneous or composite structures; surface waves; thermoelastic damping in nanomechanical resonators; and anomalous heat conduction treated via fractional calculus.