$$z'(t) = \frac{M}{t} z(t) + f(t)$$

- $\lambda = \sigma + i\varrho,\ \sigma < 0$

$$z(t) = t \int_0^1 s^{-M} f(ts)\,ds, \qquad z(0) = 0$$

- $\lambda = 0$

$$z(t) = \gamma + t \int_0^1 s^{-M} f(ts)\,ds, \qquad z(0) = \gamma = R\gamma$$

- $\lambda = \sigma + i\varrho,\ \sigma > 0$

$$z(t) = t^M \gamma + t^M \int_1^t s^{-M} f(s)\,ds, \qquad z(1) = \gamma$$

$$z'(t) = \frac{M}{t} z(t) + f(t)$$

• $\lambda = \sigma + i\rho$, $\sigma < 0$ ; $\lambda = 0$ Hcontrib.

$$z(t) = t \int_0^1 s^{-M} f(ts)\,ds, \qquad z(0) = 0$$

• $\lambda = 0$ ; $I = H + R$

$$Rz(t) = R\gamma + tR \int_0^1 s^{-M} f(ts)\,ds, \qquad z(0) = \gamma = R\gamma$$

$$Rz(0) = R\gamma$$

$$Hz(t) = tH \int_0^1 s^{-M} f(ts)\,ds \qquad Hz(0) = 0$$

• $\lambda = \sigma + i\rho$, $\sigma > 0$

$$z(t) = t^M \gamma + t^M \int_1^t s^{-M} f(s)\,ds, \qquad z(1) = \gamma$$

T1.20

$$z'(t) = \frac{M}{t} z(t) + f(t)$$

$$\boxed{Q + R + S = I}$$

• $\lambda = \sigma + i\varrho$, $\sigma < 0$; $\lambda = 0$ H contr.

$$Qz(t) = Qt \int_0^1 s^{-M} f(ts)\,ds, \qquad z(0) = 0$$

$$Qz(0) = 0$$

• $\lambda = 0$

$$Rz(t) = R\gamma + tR\int_0^1 s^{-M} f(ts)\,ds, \qquad z(0) = \gamma = R\gamma$$

$$Rz(0) = R\gamma$$

• $\lambda = \sigma + i\varrho$, $\sigma > 0$

$$Sz(t) = St^M\gamma + St^M \int_1^t s^{-M} f(s)\,ds, \qquad z(1) = \gamma$$

$$Sz(1) = S\gamma$$

$$z(t) = (Q + R + S)\,z(t)$$

It holds:

$$t^M S = S t^M, \; R = t^M R \quad \Rightarrow$$

$$z(t) = tQ\int_0^1 s^{-M} f(ts)\,ds + $$
$$t^M R\gamma + tR\int_0^1 s^{-M} f(ts)\,ds +$$
$$t^M S\gamma + t^M S\int_1^t s^{-M} f(s)\,ds =$$

$$t^M(S+R)\gamma + (Kf)(t),$$

where

$$(Kf)(t) = tQ\int_0^1 s^{-M} f(ts)\,ds +$$
$$t^M S\int_1^t s^{-M} f(s)\,ds + tR\int_0^1 s^{-M} f(ts)\,ds$$

# Boundary conditions

$$B_0 z(0) + B_1 z(1) = \beta$$

$$z(t) = y(t)\alpha + tQ\int_0^1 s^{-M} f(ts)\,ds + tR\int_0^1 s^{-M} f(ts)\,ds + t^M S\int_1^t s^{-M} f(s)\,ds \; ; \; y(t) = t^M \tilde{P}$$

$$\boxed{Q + P = I \Rightarrow QP = 0\,,\; Q\tilde{P} = 0}$$

$$\boxed{QR = 0 \quad QS = 0}$$

- $Q z(0) = 0$
- $S z(0) = 0$

$\Bigg\} \Rightarrow z(0) = R z(0) = R y(0)\alpha$

- $P z(1) = \tilde{P}\alpha + \beta_1$
- $Q z(1) = \beta_2$

$$\boxed{\Big[\underbrace{B_0 R\, y(0)}_{\substack{m\times n\; n\times n\; n\times m \\ m\times m}} + \underbrace{B_1 \tilde{P}}_{\substack{m\times n\; n\times m \\ m\times m}}\Big]\alpha = \beta}$$